Some examples

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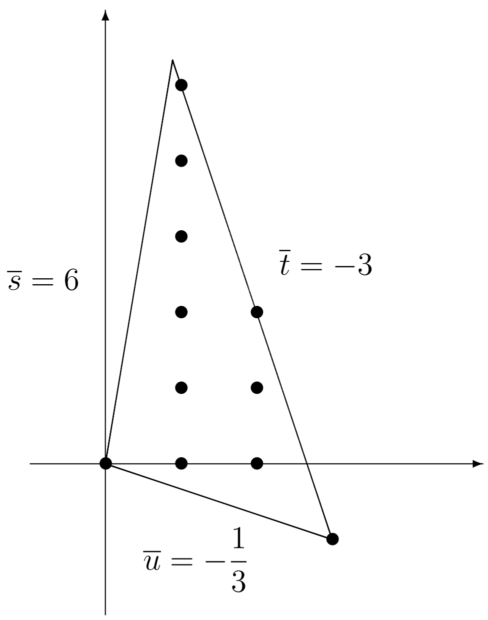
Assume $\mathrm{ch}(K) = 0$.

(1) Suppose $(a, b, c) = (8, 19, 9)$. Then,

$$s_2 = 6, \quad s_3 = 1, \quad t_1 = 2, \quad t_3 = 1, \quad u_1 = 2, \quad u_2 = 1,$$
$$\mathfrak{p} = (x^7 - y^2z^2,\ y^3 - zx^6,\ z^3 - xy)$$

and (Assumption 1), (Assumption 2), (Assumption 3) are satisfied.



$u = 3$, and

$$\ell_1 = 6, \quad \ell_2 = 3, \quad \ell_3 = 1.$$

Therefore,

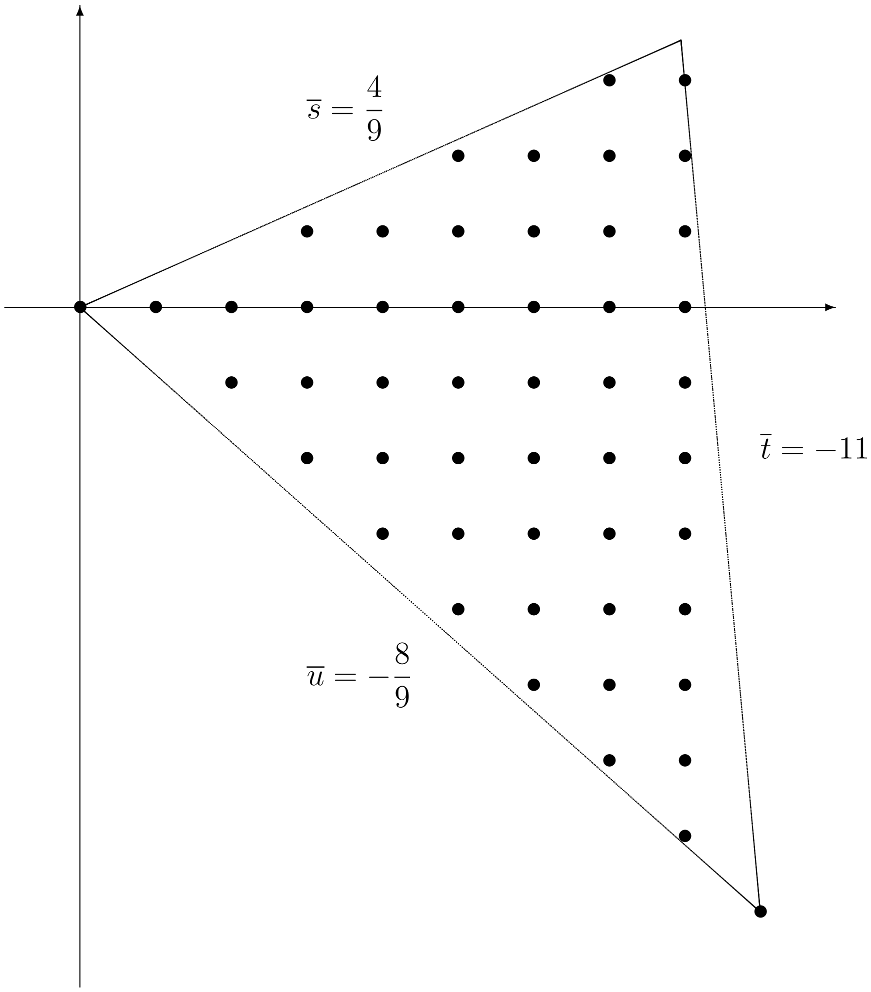
$$\ell'_1 = 1, \quad \ell'_2 = 3, \quad \ell'_3 = 6.$$

In this case, the condition EMU_1 is satisfied (Therefore, $R_s(\mathfrak{p})$ is Noetherian).

(2) Suppose $(a,b,c) = (91, 108, 103)$. Then,

$$s_2 = 4, \quad s_3 = 9, \quad t_1 = 10, \quad t_3 = 1, \quad u_1 = 1, \quad u_2 = 8,$$
$$\mathfrak{p} = (x^{13} - y^{10}z,\ y^{11} - z^8x^4,\ z^9 - x^9y)$$

and (Assumption 1), (Assumption 2), (Assumption 3) are satisfied.



$u = 9$, and

$$\ell_1 = 1, \quad \ell_2 = 2, \quad \ell_3 = 4, \quad \ell_4 = 5, \quad \ell_5 = 7, \quad \ell_6 = 8, \quad \ell_7 = 10, \quad \ell_8 = 11, \quad \ell_9 = 1.$$

Therefore,

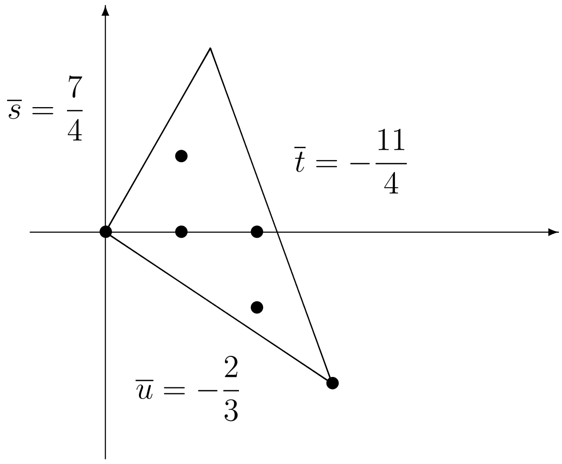
$$\ell'_1 = 1, \quad \ell'_2 = 1, \quad \ell'_3 = 2, \quad \ell'_4 = 4, \quad \ell'_5 = 5, \quad \ell'_6 = 7, \quad \ell'_7 = 8, \quad \ell'_8 = 10, \quad \ell'_9 = 11.$$

In this case, the condition EMU_1 is not satisfied (Therefore, $R_s(\mathfrak{p})$ is not Noetherian).

(3) Suppose $(a, b, c) = (25, 29, 72)$. Then,

$$s_2 = 7, \quad s_3 = 4, \quad t_1 = 7, \quad t_3 = 4, \quad u_1 = 1, \quad u_2 = 2,$$
$$\mathfrak{p} = (x^{11} - y^7 z,\ y^{11} - z^2 x^7,\ z^3 - x^4 y^4)$$

and (Assumption 1), (Assumption 2), (Assumption 3) are satisfied.



$u = 3$, and

$$\ell_1 = 2, \quad \ell_2 = 2, \quad \ell_3 = 1.$$

Therefore,

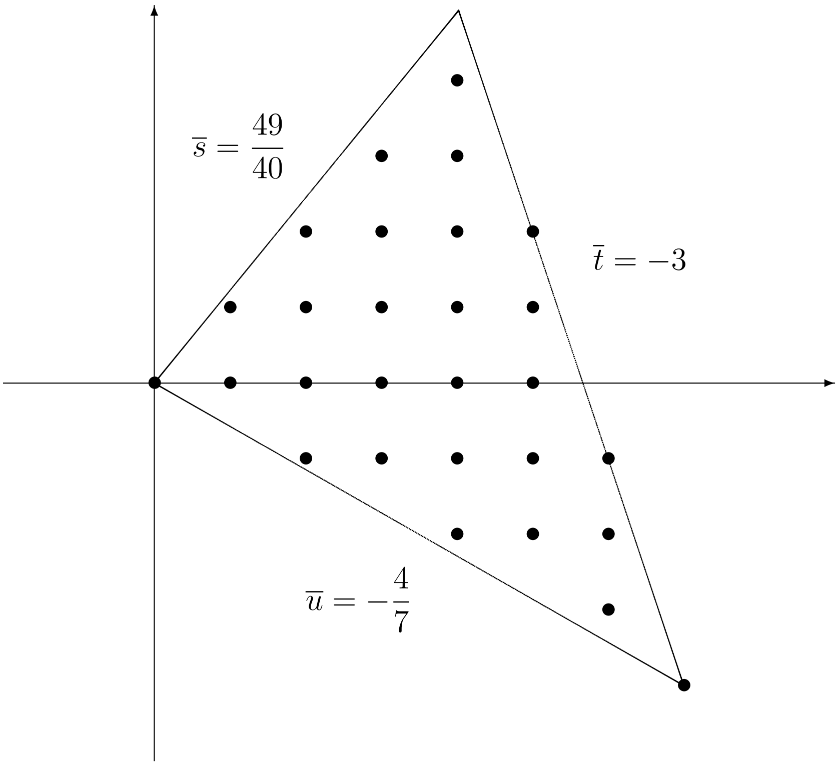
$$\ell'_1 = 1, \quad \ell'_2 = 2, \quad \ell'_3 = 2.$$

In this case, the condition EMU_1 is not satisfied (Therefore, $R_s(\mathfrak{p})$ is not Noetherian).

(4) Suppose $(a,b,c) = (17, 503, 169)$. Then,

$$s_2 = 49, \quad s_3 = 40, \quad t_1 = 2, \quad t_3 = 1, \quad u_1 = 3, \quad u_2 = 4,$$
$$\mathfrak{p} = (x^{89} - y^2z^3,\ y^3 - z^4x^{49},\ z^7 - x^{40}y)$$

and (Assumption 1), (Assumption 2), (Assumption 3) are satisfied.



$u = 7$, and

$$\ell_1 = 2, \quad \ell_2 = 4, \quad \ell_3 = 5, \quad \ell_4 = 7, \quad \ell_5 = 5, \quad \ell_6 = 3, \quad \ell_7 = 1.$$

Therefore,

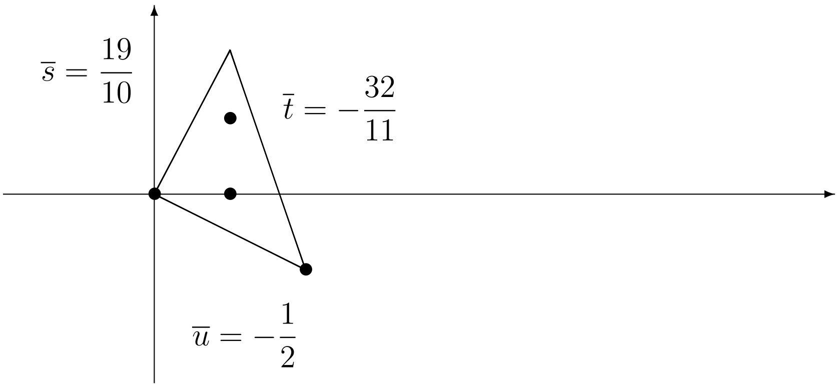
$$\ell'_1 = 1, \quad \ell'_2 = 2, \quad \ell'_3 = 3, \quad \ell'_4 = 4, \quad \ell'_5 = 5, \quad \ell'_6 = 5, \quad \ell'_7 = 7.$$

In this case, the condition EMU_1 is not satisfied (Therefore, $R_s(\mathfrak{p})$ is not Noetherian).

(5) Suppose $(a, b, c) = (53, 48, 529)$. Then,

$$s_2 = 19, \quad s_3 = 10, \quad t_1 = 21, \quad t_3 = 11, \quad u_1 = 1, \quad u_2 = 1,$$
$$\mathfrak{p} = (x^{29} - y^{21}z,\ y^{32} - zx^{19},\ z^2 - x^{10}y^{11})$$

and (Assumption 1), (Assumption 2), (Assumption 3) are satisfied.



$u = 2$, and

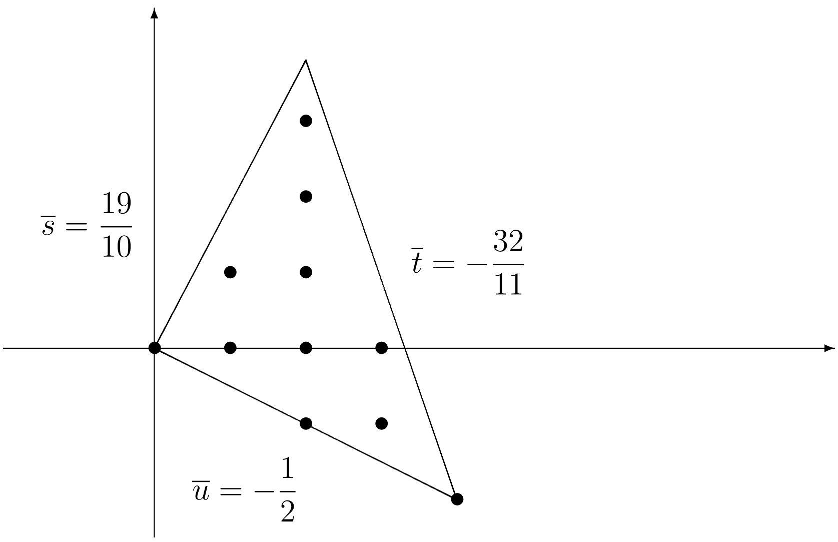
$$\ell_1 = 2, \quad \ell_2 = 1.$$

Therefore,

$$\ell'_1 = 1, \quad \ell'_2 = 2.$$

In this case, the condition EMU_1 is satisfied (Therefore, $R_s(\mathfrak{p})$ is Noetherian).

Here, consider the triangle $2\Delta_{\overline{t},\overline{u},\overline{s}}$.



$2u = 4$, and

$$\ell_1 = 2, \quad \ell_2 = 5, \quad \ell_3 = 2, \quad \ell_4 = 1.$$

Therefore,

$$\ell'_1 = 1, \quad \ell'_2 = 2, \quad \ell'_3 = 2, \quad \ell'_4 = 5.$$

In this case, the condition EMU_2 is not satisfied.